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Aehsan Haj-Yahya & Shai Olsher

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Preservice teachers’ experiences with digital formative assessment in mathematics

Aehsan Haj-Yahya a and Shai Olsher b

aThe Arab Academic Institute for Education, Beit Berl College, Kfar-Saba, Israel; bDepartment of Mathematics Education, University of Haifa, Haifa, Israel

ABSTRACT
This study investigated the experience of preservice mathematics teachers working with example-eliciting tasks (EET) on an online formative assessment platform. The study focused on the effect of working with EETs on the preservice teachers’ (PSTs) professional noticing of learners’ mathematical thinking. Participants included nine PSTs studying for their master’s degree in a teacher education college in Israel. The PSTs were presented with their own responses and peer’s responses about an angle bisector activity, and semi-structured interviews were conducted following the completion of course meetings. Qualitative methods were used to analyse the data. The findings show that the PSTs’ experiences of engagement with EETs were reflected in their noticing skills: attending, interpreting and responding. The findings also show that PSTs paid special attention to the variance between submitted examples and noticed the differences between answers. Their experiences also affected their knowledge of content and teaching, namely the proper sequence of instruction and the choice of examples for use in the classroom to facilitate deeper understanding of concepts. Finally, the findings indicate that using EETs on an online formative assessment platform enhanced the noticing skills of PSTs.

1. Introduction

One of the basic abilities in a teacher’s repertoire is pedagogical design capacity (PDC), namely, the ability to transform a written curriculum into an enacted one (Brown, 2009). This capacity is influenced by teacher’s pedagogical content knowledge (PCK) and professional noticing of learners’ mathematical thinking. Several studies have dealt with PCK and PDC (Ball et al., 2008; Boschman et al., 2014; Davis et al., 2011), as well as with professional noticing of learners (Choppin, 2011; McDuffie et al., 2018) and PDC, but few have followed the experiences of teachers as students, and the effect these experiences may have on their PDC. Researchers found that some teachers have difficulty understanding mathematical concepts (Gutiérrez & Jaime, 1999). Connecting these two lines of research, it is reasonable to ask whether teachers’ experience with mathematical tasks affects their PDC.
The present study attempts to answer this question. We investigated the possible effects that the experience of completing a set of EETs using the STEP software for online formative assessment (Olsher et al., 2016) had on PSTs’ PDC.

2. Literature review

PDC is a capacity to perceive and mobilize existing resources for the purpose of constructing instructional episodes (Brown, 2009). PDC can help explain why teachers who possess a different capacity to create and elaborate designs may enact the same curriculum differently. One of the main resources for PDC is the teachers’ knowledge. Studies show that this may take the form of knowledge of students and experience with them, the teachers’ independent learning goals (Davis et al., 2011), or their pedagogical perspective on how learning should be conducted (Boschman et al., 2014).

Some researchers have argued that it is difficult to distinguish between beliefs and knowledge (Grossman et al., 1989). Teachers’ implementation of curricula is affected not only by their knowledge, but also by their beliefs (Romberg & Carpenter, 1986). Studying the source of PSTs’ beliefs about mathematics teaching and learning, researchers have noted that for the most part these beliefs are formed during the teachers’ schooling years and are shaped by their experiences as students of mathematics or as teachers (Ball, 1988; Clark, 1988; Leinhardt, 1988; Lortie, 1975). Feiman-Nemser (1983) compared PSTs’ experience as students with the formal pedagogical education in the teachers’ education program, and argued that experiences as students have a strong effect on their conceptions about teaching.

Another factor potentially influential on teachers PDC is their ability to notice, which according to Mason (2002) has the same meaning as to perceive, and is characterized by teachers picking up ideas and trying them out for themselves. Mason (2002) describes the ‘discipline of noticing’ as teachers noticing something a colleague does, or pick up an idea from a discussion, which strikes a chord with the teacher’s own past experience, whether it fits or contradicts that experience. Jacobs et al. (2010) operationalized the concept into professional noticing of learner’s mathematical thinking by observing the mathematical strategies used by learner, interpretation of the mathematical understanding reflected in these strategies, and deciding how to respond to it in their instruction. Machalow et al. (2020) reported that some curriculum programs grant teachers opportunities to acquire noticing skills. Most teacher’s guides analysed in the study suggested evaluating students’ work without attention to their strategies, based only on procedural fluency or answer correctness, but two of the five curriculum programs reviewed frequently modelled explicit connections to conceptual student understanding. Kilic (2018) analysed preservice teachers’ written reflections, she found that the participants mostly noticed students’ errors and strategies during their interactions with students. Machalow et al. (2020) argued that teachers do not tend to use professional noticing of children’s mathematical thinking without focused education and support, and that curriculum programs should support teachers in developing professional skills for noticing mathematical thinking. Yet, improving the ability to notice classroom events, and reason about them while connecting them to broader principles of teaching and learning was achieved by professional development programs that focused on analysis of videotaped classroom events (Sherin & Han, 2004; van Es & Sherin, 2008). Uygun (2020) found that the collaborative, communicative and reflective
structure of StoryCircles digital technology increased the participants’ levels of noticing student mathematical thinking. Professional noticing, together with teacher’s orientations, was associated by McDuffie et al. (2018) with teacher’s design of curricular materials. Choppin (2011) found that teachers who attended closely to student thinking developed conjectures about how that thinking developed, and related their conjectures to the details of student strategies, leading to adaptations that enhanced task complexity and students’ opportunities to engage with mathematical concepts.

Examples play an important role in giving meaning to mathematical definitions. Examples are usually not isolated, but rather perceived as instances of a class of potential examples, constituting an example space related to a particular concept, at a particular time, in a given context (Watson & Mason, 2006; Zaslavsky, 2014). Different students have different example spaces for the concept familiar to them (Vinner & Hershkowitz, 1983).

A common phenomenon that leads to many mathematical mistakes has to do with prototypical examples. There is at least one prototypical example for every concept in geometry (Hershkowitz et al., 1990; Vinner & Hershkowitz, 1983). The prototypical examples are acquired first, and are therefore found in the concept image of most learners. Prototypical examples usually are a subset of examples of the concept, and have the longest ‘list’ of attributes, including critical attributes of the concept, and some that are specific to that subset but not critical. The non-critical attributes have dominant visual properties, affect the construction of geometric concepts, and affect the abilities of the student to identify, classify, construct and make judgments about basic concepts in geometry (Hershkowitz, 1990).

Research revealed common mistakes made by secondary school students (Vinner & Hershkowitz, 1983) and preservice elementary teachers in the concept of altitude in triangles (Gutiérrez & Jaime, 1999). These errors are due to the mistaken, incomplete concept image of the altitude in a triangle. For many participants, the easiest altitude to construct is one that is internal to the triangle, as in the case of acute triangles. Altitudes were more difficult to construct in obtuse and right triangles, which have non-internal altitudes. Another factor affecting an incomplete concept image of altitude was the position of the triangle: those in a prototypical position, with a vertical altitude, were easier to construct. The position of the triangle affected the number of wrong altitude constructions. Gutiérrez and Jaime (1999) found that the most frequent errors were caused by confusing two concepts: the altitude and the median, or the altitude and the perpendicular bisector. These errors may originate in the prototypical concept image of altitude, which in an isosceles triangle is drawn from the top vertex to the base, and serves also as the angle bisector and median to the opposite side. Research is still lacking on the concept image and concept definition of the other cevians: the median and angle bisector.

PCK, the special domain of teachers’ knowledge, was first coined by Shulman (1986). Ball et al. (2008) investigated the content knowledge needed for teaching, specifically in the domain of mathematics, and distinguished four types of knowledge, two of which are closely related to pedagogic design capacity: knowledge of content and students (KCS) (e.g. knowing what students find easy or difficult, what motivates them and what they are likely to find boring) and knowledge of content and teaching (KCT), which is required for instructional design, and includes teaching sequence, choice of examples for introducing a topic, choice of examples for deepening the students’ understanding, and more.
These components of knowledge of the psychological aspects of the example space, the prototypical examples of mathematical concepts, and in particular, of the cevians in triangles, are necessary for professional noticing, which combines attending to the mathematical strategies used by learners, interpretation of the mathematical understanding reflected in these strategies, and deciding how to respond in their instruction (Jacobs et al., 2010).

Professional noticing is synonymous in many ways with formative assessment (FA), defined by Black and William (2009) as ‘evidence about student achievement… elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited’ (p. 7). Black and William (2009) specified five key FA strategies: clarifying and sharing learning intentions and criteria for success; engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding; providing feedback that moves learners forward; activating students as instructional resources for one another and activating students as the owners of their own learning.

STEP is an online FA platform designed to support the assessment of inquiry-based learning and various patterns of open-ended EETs (Olsher et al., 2016), including interactive diagrams in a dynamic geometry environment (DGE). Such diagrams provide opportunities for making and testing conjectures, performing investigation and conducting inquiry-based learning activities, which are encouraged by the mathematics education community (Chiu & Churchill, 2015).

STEP allows teachers to choose characteristics and filter student answers based on these characteristics, enabling teachers to design effective learning tasks that elicit evidence of student understanding and stimulate dialogue in the class, as part of FA (Black & William, 2009). The vast majority of activities within STEP include EETs that require students to create and submit examples or non-examples to support or contradict a mathematical claim, or to create examples under given constraints. The analysed student work on these tasks enables the teacher to gain insight about the students’ example space for the given mathematical concept, and to determine whether it includes only prototypical or a wider range of characteristics (Hershkowitz et al., 1990; Vinner & Hershkowitz, 1983; Watson & Mason, 2006; Zaslavsky, 2014).

3. Rationale and research questions

Review of the literature shows little evidence of the effect of PSTs’ experience as students on their professional noticing of learners’ mathematical thinking. The present study begins to fill this gap. We investigate whether a PSTs’ experience in performing mathematical tasks and analysing their collective answers using an online FA platform (STEP) is reflected in their professional noticing of learners’ mathematical thinking.

4. Methods

In the current study, we used qualitative methods, after a group of PST’s performed an activity on an FA platform on the topic of angle bisector they were invited for a semi-structured interview. During the interview the complete grid of answers pictures of the whole group of PST’s was presented to them anonymized. The PST were able at most
cases to identify their own answers, but could look at and evaluate with a comparative and criticizing glance their own and their peer’s responses. This is the focus of this study.

4.1. Research setting

The study was conducted during the 2017–2018 academic year, as part of a didactics of teaching mathematics course for PSTs studying for their M. Teach (Master of teaching) degree at a teacher education college in Israel. The two-year program is designed to provide master’s level training for candidates with a mathematics or engineering background, and high academic and professional skills, to obtain a certificate for teaching secondary school. In addition to supplementary mathematics courses, the curriculum focuses on three important facets that aim to promote teaching-learning-assessment in action in the field of education: (a) learning and thinking, planning and evaluation; (b) social aspects of the teacher’s work: diversity and equality, society and community; and (c) valuable aspects of teaching: profession and ethics, aesthetics and education.

The main didactic aims of the teaching mathematics course were to enhance the prospective teachers’ content knowledge and psycho-didactic knowledge. Special emphasis was placed on various aspects that characterize mathematical activity: the procedural, formal, and intuitive aspects, as well as the connections between them. The course addressed methods of teaching mathematics based on the research literature and on analysis of events from the field, observed by PSTs in their in-class training. The goals of the course were to train mathematics PSTs, with emphasis on cultivating pedagogical, curricular, and didactic knowledge. As part of the course, PSTs were exposed to technological tools in mathematics education in general (12 h), and to the STEP platform (Olsher et al., 2016) for formative assessment in particular (8 h).

The course work with the STEP platform included four meetings of two academic hours each. The first two meetings were devoted to theoretical background on FA and an introduction to the student interface of the STEP platform, in which the PSTs completed two tasks: one in calculus and one in geometry (see Appendices 1 and 2). The third meeting included an introduction to the STEP teacher interface, with an analysis of a set of answers by PSTs to the two introductory tasks that they had completed at the previous meetings. PSTs were asked to identify the characteristics of their responses and suggest criteria for categorizing them. Following their suggestions, the lecturer (the first author) discussed with the PSTs the characteristics that were analysed automatically by the STEP platform. During the fourth meeting, the PSTs solved the angle bisector activity, completing the tasks in the student’s interface. Following the four meetings, PSTs were interviewed about the submitted solutions (their own and their peer’s) and about the use of the activity in teaching.

4.2. Participants

Nine mathematics PSTs, 6 weeks into their first semester in the first year of study for the M. Teach (Master of teaching) degree at a teacher education college in Israel participated in the study. During that semester, in addition to the didactics of teaching mathematics course, the PSTs participated the courses in guided experience in mathematics teaching, statistics in teaching, qualitative methodology in education, diversity among learners and
Table 1. Education of the PSTs.

<table>
<thead>
<tr>
<th>Field</th>
<th>B.A. degree</th>
<th>M.A. degree</th>
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<tbody>
<tr>
<td>Computer engineering</td>
<td>3</td>
<td>1</td>
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<tr>
<td>Electronics engineering</td>
<td>2</td>
<td>–</td>
</tr>
<tr>
<td>Business administration</td>
<td>1</td>
<td>–</td>
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<tr>
<td>Economics</td>
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In the first semester, the course in the didactics of teaching mathematics was the only pedagogical course offered to PSTs. PSTs had either a bachelor’s or a master’s degree in mathematics or a closely related field (e.g. engineering) from an Israeli university. Some had degrees in computer engineering, electronics engineering, economics or business administration (Table 1). On average, about 12 years had passed since they completed their previous academic studies. All had worked in their professions for several years, and none of them had any teaching experience before entering the M. Teach program. The program granted a master’s degree and an Israeli secondary school mathematics teaching certificate.

4.3. Research tools

The research tools used in the study included: (a) the STEP angle bisector activity, which was presented to PSTs, and (b) the set of PSTs’ answers, with all the responses of the PSTs to the tasks of the activity. The names of the PSTs were deleted from the sets of answers that were later presented to them during the interview.

The angle bisector activity was designed to enable the assessment of students’ use of the angle bisector theorem and to advance the understanding of the various components of one of its possible proofs: necessary and sufficient initial conditions of the theorem and the underlying principle of equal distances of rays forming an angle from each point on the bisector of the given angle. An additional goal was to assess the depth of the concept image PSTs have of the angle bisector.

The activity included three tasks (Figure 2), based on design patterns of EETs requiring students to submit multiple examples that can be automatically analysed (Yerushalmy et al., 2017).

For the construction shown in Figure 1, statement ii is always true because triangles ADC and ADB have a common altitude from point A. Statements i and iii are equivalent, and they are true when cevian AD is an angle bisector. Thus, for the first task the answer is yes; statement ii should be chosen, and the submitted examples should provide triangles for which AD is not the angle bisector. For the second task, the answer is no, because the first and the third statements are equivalent. For the third task, the answer is yes, and the submitted examples should contain triangles for which AD is an angle bisector for angle A. An additional goal of the third task is to elicit examples other than prototypical examples of bisectors (e.g. a bisector of the vertex angle of an isosceles triangle). This activity enables us to investigate and to advance mathematical thinking. It helps us see the PSTs’ concept image of the angle bisector when they are asked to submit three different examples. The teacher can consider the collage of answers and determine based on the filters whether the learners submitted only prototypical examples (Hershkowitz et al., 1990). The activity
emphasizes the definition of angle bisector as a locus (set of points whose location is at the same distance from the angle rays). PSTs can construct three different triangles when all the statements are true if and only if AD is an angle bisector. One of the statements is that the altitudes (distance) from point D (DG, DH) to the sides AB and AC (the angle rays) are equal, and when these altitudes are equal, the ratio between the areas of the triangles ABD and ACD is the same as the ratio between the sides AB and AC, when calculating the area of the triangles using the formula $\frac{S_{\triangle ABD}}{S_{\triangle ADC}} = \frac{(AB \cdot DH)}{(AC \cdot DG)}$. By analysing these characteristics, the PSTs may be able to identify which submissions contained only prototypical examples of angle bisectors, and makes it possible for the PSTs (in their role as teachers) to address the difficulties of the learners that submitted such examples, and to present these examples to the entire class for discussion.

The activity emphasized two meta-pedagogic mathematic issues: (a) the need for proof when the PSTs discover that two statements are equivalent (the first and the third statements); and (b) providing scaffolding for PSTs to prove the angle bisector theorem (if AD is an angle bisector, then $\frac{AB}{AC} = \frac{BD}{DC}$); this theorem is used extensively in the geometry curriculum in secondary school. The second statement is always true because of the common altitude from vertex A; if AD is an angle bisector, it follows from the definition of the angle bisector that the altitudes DG and DH are equal (first statement), which leads to the third statement because of the equivalence between the first and third statements.
4.4. Data sources and analysis

The data sources included: (a) the STEP data generated by the angle bisector activity, including the various PST answers collected and their automatic analysis, and (b) semi-structured interviews with the nine PSTs. The interviews lasted about 15–20 min and were conducted 3 weeks after the PSTs completed the four course meetings. PSTs were presented with the PST answers and asked three questions: (a) ‘What characteristics of the PSTs’ work are evident from their answers to the activity, and how would you classify the answers according to these characteristics?’ This question sheds light on two components of the professional noticing of learners’ mathematical thinking. The ability to attend to the mathematically important details of learners’ strategies, and the ability to interpret the mathematical understandings reflected in these strategies. The aim of the question is to reveal whether the PSTs made sense of the details of each strategy, and whether they noted how these details reflected what learners understand. Second, the PSTs recognized what strategies and understandings the learners failed to attend. In analysing the PSTs’ assessment of the learners’ performance, we were not seeking a single best interpretation of that performance, but were interested in the extent to which PSTs’ reasoning was consistent with the details of a given learner’s strategies and understanding, and with the research on learner’s mathematical development. (b) ‘How would you use this activity as a teacher?’ and (c) ‘What was the original goal the designers intended for this activity?’ The second and third questions reflect the third component of noticing, deciding how to respond which reflects the decisions PSTs make for instruction. Responses demonstrate that PSTs interpreted the students’ understanding by trying to make sense of strategy details. PSTs’ responses can be of various types, we chose to focus on the reasoning involved in selecting the next problem for the classroom. We were not seeking a particular next problem or rationale, but were interested in the extent to which PSTs based their decisions on what they had learned about the learners’ understandings from the specific situation, and how consistent their reasoning was with research on learners’ mathematical development.

We divided the interview transcript into episodes by questions. For each episode, we analysed the data using qualitative methods: coding and categorizing. We used codes and categories that we derived from previous studies; additional codes were derived inductively from themes that appeared in the data (Charmaz & Belgrave, 2007; Saldana, 2015). Next, we grouped the codes under the suitable categories based on a ‘coding scheme’ (Lewins & Silver, 2007). For example, by the interview question ‘What characteristics of the PSTs’ work are evident from their answers to the activity, and how would you classify the answers according to these characteristics?’ we referred to the first and second components of noticing, which are attending and interpretation, and also to the teachers’ KCS. We constructed two categories: correctness and variance. Under the correctness category we placed two codes: correct answer to the question (preconceived code) and matching examples and responses (preconceived code). Under the variance category we placed the following codes: type of triangles by sides (preconceived code), type of triangles by angles (preconceived code), position of angle bisector (preconceived code), position of triangles (inductive code) and size of triangles (inductive code).

The process of analysis was iterative, matching each relevant episode into a category that has certain characteristics of the PSTs’ practice with the definitions of mathematical concepts. We identified the categories described in the Results section.
Below we present three PSTs, whose responses and interviews are representative of the PSTs, in that they include all the categories and codes that we constructed before when analysing the nine interviews.

5. Results

Regarding the first interview question, our analysis revealed two main categories of characteristics that PSTs assessed and interpreted, referring to in the PSTs’ work: correctness of the submitted answers and variance within submitted examples.

Regarding the second and the third interview questions, our findings revealed discrepancies between the ways in which PSTs described the goals and the possible solutions of the activity. These discrepancies are related to their experience of solving the tasks in the students’ interface and analysing their collective answers. We present the different goals described by the PSTs and suggest possible connections to their experiences with the activity, then analyse their perspective about possible use of the activity from the point of view of FA, for the purpose of conducting an effective classroom discussion.

5.1. Characteristics used to classify PSTs’ answers, as described by their colleagues

Analysis of the PSTs’ responses, with reference to the characteristics evident in the PSTs’ work, and the grouping of these characteristics reveal two categories that attracted PSTs’ attention: correctness and variance between the three submitted examples. The category of correctness was interpreted based on two characteristics: choosing the correct constraints and correctly matching the examples submitted with the constraints chosen. With respect to variance, five categories were identified and interpreted as enhancing variance: (a) type of the triangles based on sides (e.g. isosceles, equilateral, scalene); (b) type of the triangles based on angles (e.g. acute, right, obtuse); (c) position of the triangles; (d) position of the angle bisector and (e) size of the triangles (side length).

Below we describe each type and subtype, and illustrate it with the PSTs’ responses.

5.1.1. Correctness

PSTs’ level of elaboration and interpretation of what correctness meant to them were varied. When asked how he would sort the PSTs’ answers, Avi (pseudonyms are used for all PSTs) stated that ‘First, based on whether the answer is correct’, without specifying what a correct answer is. Another PST, Shiri, identified correctness as follows: ‘I would check for a match’, without specifying whether she would check that the constraints chosen were correct. Only one PST, Chen, explicitly stated that he would initially look for correctly chosen constraints and matching examples. Chen’s answer concerning the third sub-task (Figure 1) illustrates this:

Author: Take the third sub-task … How would you sort out your colleagues’ answers?

Chen: I would classify who answered correctly and who answered incorrectly.

Author: Is this the first classification?

Chen: The second classification is correct examples that match the answer, and variety. After all, he has a correct answer, three different forms of triangles.
5.1.2. Invariance

When considering PSTs’ answers, PSTs did not restrict themselves to the correctness of the submitted answer but also addressed similarities and differences between the answers, which were unrelated to errors or level of correctness, to identify what strategies and understandings the learners demonstrated or failed to demonstrate. PSTs interpreted the variance of the examples based on five criteria listed above. Below we describe how each characteristic was presented by the PSTs, and provide examples of PSTs’ work illustrating each characteristic.

5.1.2.1. Type of the triangles by sides.

One way of differentiating between triangles is by categorizing them according to the relations between their sides: isosceles, equilateral and scalene. In task 3 (Figure 1), the required cevian is the angle bisector. The prototypical concept image of the angle bisector is that an isosceles triangle, with the bisector dropping from the top vertex to the base. PSTs showed awareness of this phenomenon, as shown in the interview with Chen:

Chen: Here it is also beautiful [referring to the submitted answer shown in Figure 2]

Author: Why beautiful?

Chen: As I said before, the three types of triangles are identical. You want to see three examples of different triangles that are special and not special… Here you see the same thing [referring to the submitted answer shown in Figure 3], and also here the same thing [referring to the example on the right].

Author: What is special about the examples of student 7 [in Figure 3]?

Chen: All of them are isosceles … This could be my submission.

Chen referred to the submission shown in Figure 3 as 'beautiful'. It includes angle bisectors constructed from the top vertex to the base of an isosceles triangle, and in two scalene triangles, acute and obtuse. Chen referred to an answer that included only the prototypical example of an isosceles triangle, with the angle bisector dropping from the top vertex to the base, as a triplet of similar examples (although one of the triangles is obtuse and the others are acute), contrasting the answer with the ‘beautiful’ example that has scalene triangles in it. Chen described the details of two answers (Figures 2 and 3), explaining how
these reflected what the learners understood and what they did not, based on his assessment of whether the PST submitted only prototypical examples of angle bisectors or also non-prototypical ones. This is an example of interpretation of mathematical understanding based on which examples the PSTs submitted and which they did not. Note that Chen referred to the triplet in Figure 3 as possibly his own, providing some indication that his experience in the student interface may have affected the lens through which he analysed or filtered the PSTs’ responses.

5.1.2.2. Type of triangles by angles. Another way of differentiating between triangles is to categorize them based on their largest angle: acute, right, or obtuse. Shiri used this characteristic when classifying PSTs’ answers to tasks 1–3. She chose this characteristic as the one that produced the greatest diversity between the answers of most PSTs:

Shiri: As we said in previous cases, we see how different the examples are from each other.

Author: Please look at the examples. Which are interesting and which are not?

Shiri: This one gave the most diverse examples. It has a right angle triangle, an acute triangle, and an obtuse triangle.

5.1.2.3. Position of the triangles. In task 1 (Figure 1), the required cevian is not the angle bisector (the only case in which only one condition can be met). When analysing PSTs’ answers, the PSTs showed awareness of the position of the triangles in the triplet, and of the fact that all of them had sides that were aligned the same way, as shown in the interview with Avi:

Author: What did we say the correct answer is here?

Avi: The correct answer is yes [it can be met], and… wait a minute. Here I must have them [the triangles] not identical. I mean I must have them not-identical… It’s fine [referring to the top triplet shown in Figure 4], but there is no diversity in the examples… He’s supposed to give three examples. I think he only moved the point [point A], but they are from the same family, these examples.

Author: What do you mean by the same family?

Avi: The family that he submitted. In these examples, DG is longer than DH… but the examples look very similar… [referring to the bottom triplet shown in Figure 5] Here they are more elegant, I don’t know if it’s more elegant, but he did something interesting, he rotated [the triangle].

Avi looks for variance between the examples, and whether they belong to ‘the same family’, suggesting that rotating the triangles creates more ‘interesting’ and diverse examples.

5.1.2.4. Position of the angle bisector. As noted, the prototypical concept image of the angle bisector is that of the bisector dropping from the top vertex to the base. The PSTs used this aspect as a criterion for categorizing the variance between examples submitted by the PSTs. The following excerpt is from the interview with Chen, when he was asked to provide criteria for the classification of PSTs’ answers to task 1 (Figure 1):

Chen: Here it’s beautiful.
Author: The fourth?
Chen: Yes.

Author: Why is it beautiful?

Chen: As I said before, first of all three types of triangles [referring to the triplet shown in Figure 5] ... but it would be smarter if the right angle [from which the angle bisector is constructed] weren’t up but down.

Chen added the position of the angle bisector as a criterion that increases the variance between examples. He referred to examples in which the angle bisector is not constructed from the top vertex of the triangle as ‘original’.

5.1.2.5. Size of the triangles. Some PSTs considered the size and the appearance of the triangle as a factor in the variance of the examples. When analysing the answers to task 3, Avi checked for different lengths of the sides of the triangles:

Avi: I hope the length is different here ... I think it’s different. It’s the same here ... very similar examples. Here I see a different size of triangle. It is rotated. So it seems that the fifth [triplet shown in Figure 6] looks fine and a little varied.

Avi looked at size and shape as criteria for variance, although referring to the triangles as only ‘a little varied.’
Figure 6. Triplet of examples submitted for task 3.

The different interpretations of variance reflected the mathematical understanding of the concept of angle bisector. Shiri mentioned the type of triangle based on angle, Avi the position of the triangle, and Chen the position of the angle bisector as their criteria for variance. Based on these interpretations, they determined which examples reflected mathematical understanding of the angle bisector concept and which did not, identifying non-prototypical examples in the PSTs’ answers. All of them brought evidence consistent with their interpretation of variance.

5.2. Purpose of the activities and how PSTs intended to use them (PDC)

PSTs’ experience working with the activities in the students’ interface and analysing their collective answers affected their perception of the goals of the activities and their choice of possible uses of the activities to promote learning.

5.2.1. Goals of the activity

PSTs were asked how they can use the activity as teachers. Chen and Avi perceived the goals of the activity as deepening the understanding of the angle bisector concept and connecting it with other relevant concepts in the angle bisector theorem. Shiri, however, suggested that the goal of the activity was to help students understand the angle bisector concept itself, and provide a context for the angle bisector theorem. In the following section, we describe the two different goals.

Avi: The goal is to sharpen the understanding of the connection between concepts in geometry, that is, the relation between triangle similarity, angle bisectors, ratio between angles, medians... What leads to what and what is separate from what. It sharpens and leads to a deeper understanding. Perhaps it causes better intuitions... The goal of the angle bisector activity is perhaps to examine the perception... possible mistakes in geometry, and what is their origin... Maybe there is a connection with the prototype of the shape... Of perception of certain mistakes.

Avi emphasized the fact that the activity can help develop a more precise and accurate concept image of the angle bisector, beyond the prototypical one. He also noted that the prototypical concept image may have produced some errors. Avi’s arguments are consistent with clarifying and sharing learning intentions and criteria for success, one of the five key FA strategies (Black & William, 2009).

Chen stated that ‘The goal, in my opinion, was to sharpen the characteristics of... the relationship between the characteristics of the triangle... angle bisector, and altitude...’ Similarly to Avi, he also addressed the connection between different characteristics of the
triangle and concepts related to it as the goal of the activity. The sharpening is achieved by linking the concept of angle bisector with other concepts, such as the altitudes.

According to Shiri, the goal was to accomplish initial learning and understanding of the concept of the angle bisector, rather than clarifying this understanding. Encouraging students to assume ownership of their own learning is one of the five key FA strategies (Black & William, 2009). Shiri said:

Shiri: The goal, as I saw it initially, was to understand the characteristics of the angle bisector… starting with what is an angle bisector. Because, as I recall, there were points referring to … you had to notice that the angle is equal [to the other part of the angle]. When it bisects it's equal. You can see that you know and understand the angle bisector theorem and its meanings in terms of the relationship between the sides, thus seeing that the topic of the angle bisector in the triangle is clear … I think it was too complicated for the PSTs. From my personal experience, it was exhausting … it was a bit exhausting and a bit frustrating.

From Shiri’s perspective, the activity would be difficult for her potential students. A possible explanation is that she found it difficult to solve the problem, demonstrating a CCK that is not deeper than that of the students themselves. Based on her level of CCK, she found the activity complicated, exhausting, and frustrating. She thought that the goal of the activity was to help students understand the angle bisector concept and the related theorem, which is one component of the KCT.

5.2.2. PSTs’ responses that promote learning
PSTs were asked to choose responses for task 3 (Figure 2) that they believed would promote the learning process, that is, to select the next problem in classroom. This represents the third component of noticing: responding. Chen’s and Avi’s choices emphasized the non-prototypical angle bisector position and the triangle position:

Chen: The first and last … [2 triplets shown in Figure 7]

Author: Why?

Chen: The first one brings three different types.

Author: Is this the first answer you would show on the board?

Chen. To notice that each type of triangle, not [only] special [types of triangles], is different produces the same result, but I would also add PST 8 [Figure 8], who has a right angle triangle here but not at the top [vertex].

Chen chose to present different examples, based on the triangle angles (acute, right and obtuse), which differ from the prototype (isosceles triangle with the angle bisector constructed from the top vertex). Chen proposed to expose students to other examples, by enriching the example space for the angle, to expand the concept image of the angle bisector. The choice of non-prototypical examples, which was expected to motivate students to think out of the box, demonstrates Chen’s KCS. By choosing to use this activity to sharpen the concept of the angle bisector, Avi and Chen demonstrated similar components of their KCT. Their aim was to ask learners to compare examples in order to challenge their thinking, and they used answers that include variance in the examples submitted, based on their interpretation about variance. Shiri chose a different way to engage her students in the discussion:

Shiri: I’m taking the third PST’s answer [triplet shown in Figure 9].
Author: Why?
Shiri: Because it’s the same, it’s isosceles.

Author: And how would you start the discussion?

Shiri: I would tell them that this is a student’s answer and ask the students to check the correctness of the answer … Then, if they said it was correct … I would ask them what they thought about the answer in terms of its quality.

Author: What do you mean by its quality?

Shiri: To create for them some kind of distinction … take them out of the way kids think, test type of thinking … right or wrong. Right gets a good grade and wrong doesn’t. Because it’s not a test, you can give a correct answer that can have its qualities and variety … It means that the answer is correct and you get all the points, but maybe there’s a better answer that explains more or is more diverse, more in that respect.

Shiri’s choice of how to proceed was affected by her interpretation of variance. Her response included three prototypical examples of the angle bisector drawn from the vertex to the base of the isosceles triangle. She also encouraged discussion by asking about the quality of the responses. Shiri thought that by choosing prototypical examples and asking about the quality of the response she could challenge and motivate the students to expand their concept image of the angle bisector, which shed light on one component of her KCS.
6. Discussion

This study investigated whether PSTs’ experience performing an activity with EETs, and subsequent use of their collective answer space in an online FA platform (STEP), are reflected in their references to the richness of the students’ example spaces and in their PDC. We sought to answer these questions by inquiring how PSTs would use this activity in their teaching.

We can see that the activity may provide PSTs an opportunity to learn noticing skills (Machalow et al., 2020; Uygun, 2020). Uygun (2020) found that using StoryCircles digital platform supported the participants’ noticing of student mathematical thinking. The findings show that PSTs paid special attention to the variance between submitted examples, compared with the research of Kilic (2018) on which the preservice teachers mostly noticed students’ errors and strategies during their interactions with students. They made distinctions based on a criterion they chose to assess the variety of examples (Mason, 2002). PSTs paid attention to the mathematically important details of the learners’ examples space, which could inform a teacher’s instruction. They identified and focused on ‘noteworthy aspects of complex situations’ and discerned the patterns of answers (Jacobs et al., 2010) about the angle bisector. They sought to find out whether the example space of the angle bisector (Watson & Mason, 2006; Zaslavsky, 2014) was comprehensive, and whether it comprised any examples in addition to the prototypical ones (Hershkowitz et al., 1990). PSTs evaluated learners’ mathematical thinking and interpreted students’ understandings, which were reflected in their colleagues’ answers (Jacobs et al., 2010), by referring not only to the correctness of the answers, but also to the learners’ common conceptions and misconceptions as they grappled with the angle bisector theorem (Ball et al., 2008). PSTs emphasized that students might conclude that the angle bisector must be prototypical, and checked whether the examples submitted had prototypical characteristics. To differentiate between student examples, PSTs suggested grouping the examples into categories based on several criteria: type of triangles by sides (isosceles, equilateral, scalene) to identify students who tend to construct the angle bisector in isosceles triangles; type of triangles by angles (acute, right, obtuse), to identify students who tends to construct the angle bisector in acute triangles; position of the triangles, to identify students who tend to construct the
angle bisector in triangles in which one side is horizontal; and position of the angle bisector, to identify students who tend to construct the angle bisector from the top vertex. PSTs sought to categorize the responses using almost the same criteria that were suggested by Gutiérrez and Jaime (1999) for analysing PSTs’ conceptions of the altitude. This behaviour attests to the ability of the PSTs to interpret the understandings reflected in the collection of submissions. PSTs grasped the details of each strategy and noted how these details reflected the strategies. PSTs also recognized what strategies and understandings their colleagues did not adopt (Jacobs et al., 2010).

The findings also show that the experience of PSTs with EETs using the STEP platform may increase their noticing ability (Jacobs et al., 2010; Mason, 2002). PSTs emphasized all three component skills of professional noticing of students’ mathematical thinking: attending to mathematical understanding reflected in the learner’s strategies, interpreting the mathematical understandings reflected in these strategies, and responding to these understanding based on this interpretation. Teachers’ PCK, namely KCS and KCT (Ball et al., 2008), was expressed in the PSTs’ noticing and pedagogic design capacity. KCS was manifested in knowledge about examples that are easy, difficult, confusing, and motivate learning. PST helped categorize the examples submitted into those that are easy to identify and construct (prototypical examples), and those that are difficult to identify and construct. The platform provided PSTs with the opportunity to develop the skills of attending and interpreting their colleague’s answers, and the opportunity to decide how to respond, providing evidence of interpretation of the PSTs’ understanding (Choppin, 2011; Jacobs et al., 2010). PSTs’ responses attest to their KCT, which included sequencing of instruction and examples to be used in the classroom that could lead to deeper understanding of the concept. Boschman et al. (2014) noted that the teachers’ beliefs and convictions play an important role in their PDC. In our study this was illustrated by one PST proposing to challenge her students with the quality of an answer by showing them examples that are quite similar, whereas the other PSTs preferred a response that contained different examples to achieve the same goal.

Confirming the findings of Leinhardt (1988), our study showed that the PSTs’ experience in the students’ interface affected their conception of the goals and use of the activity. One PST (Shiri) found the activity to be difficult and complicated, suggesting that its goal and utility was to help students learn and understand the angle bisector concept and theorem. This could be a result of her own difficulty to recognize the attributes of the angle bisector and to make the connection between them. The PST who submitted only prototypical examples (Chen) suggested that the goal and utility of the activity was to clarify and expand the angle bisector concept and theorem, demonstrating the effect of this experience on his objective in using the activity.

To provide additional information about preservice and in-service teachers’ pedagogical knowledge development in similar and other mathematical contexts, further studies are needed, with a larger and more diverse research population that includes in-service mathematics teachers. This could be achieved by following the in-service teachers’ experience with the student’s interface, classroom observations of lessons, and the design of lesson plans. The fact that up to the intervention described in this study there were no pedagogical courses offered to the PST’s, leads us to believe that the intervention we conducted, aimed to achieve this type of result was related to the change. Yet the fact that we aimed
to minimize other factors which might cause this change, is not enough to tie this change only to this brief intervention, and should be further studied.

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No potential conflict of interest was reported by the authors.

ORCID
Aehsan Haj-Yahya http://orcid.org/0000-0002-5495-7065

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